

Quark and Lepton Mass Patterns and the Absolute Neutrino Mass Scale

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We investigate what could be learned about the absolute scale of neutrino masses from comparisons among the patterns within quark and lepton mass hierarchies. First, we observe that the existing information on neutrino masses fits quite well the unexplained, but apparently present regularities in the quark and charged lepton sectors. Second, we discuss several possible mass patterns, pointing out that this is consistent with hierarchical neutrino mass patterns especially disfavoring the vacuum solution.

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Non-vanishing neutrino mass squared differences imply neutrino oscillations, which in fact have been observed in recent years. The measurements of the mass squared splittings between the mass eigenstates ν_l , $l = 1, 2, 3$, give the hierarchy $\Delta m_{21}^2 \equiv m_2^2 - m_1^2 \ll \Delta m_{32}^2 \simeq \Delta m_{31}^2$. Atmospheric neutrino oscillations are dominated by the large mass squared splitting $\Delta m_{32}^2 \simeq \Delta m_{31}^2 \simeq 3.3 \cdot 10^{-3} \text{ eV}^2$, while solar neutrino oscillations allow four different solutions. These are the so-called LMA, SMA, LOW, and VAC solutions with $\Delta m_{21}^2 \simeq 3 \cdot 10^{-5} \text{ eV}^2$, $7 \cdot 10^{-6} \text{ eV}^2$, 10^{-7} eV^2 , 10^{-10} eV^2 , respectively [1, 2, 3], where the LMA solution is preferred after inclusion of the latest SNO data. So far, for the absolute neutrino mass scale only upper bounds from several experiments exist (for an overview see, e.g., Ref. [4]): The kinematical endpoint of tritium beta decay leads to $m_1 \leq 2.2 \text{ eV}$, while $0\nu 2\beta$ -decay (neutrinoless double beta decay) even implies a stronger bound for the electron neutrino Majorana mass, $\nu_e, m_M^{\nu_e} \leq 0.2 \text{ eV}$. Furthermore, somewhat weaker but similar bounds emerge from astrophysics and cosmology. However, except from these bounds, the absolute neutrino mass scale is not yet known. Thus, in the most extreme cases, hierarchical ($m_1 \ll |\Delta m_{21}| \equiv |m_2 - m_1|$) or degenerate ($m_1 \gg |\Delta m_{21}|$) mass spectra are allowed, which ultimately should be understood in some theoretical model. In this paper, we will observe that neutrino masses fit the well known empirical regularities of quark and charged lepton masses. We will generalize this discussion and use rather simple models and assumptions in order to obtain information on the absolute neutrino mass spectrum from a phenomenological comparison with the quark and charged lepton mass spectra. This implies that we argue in terms of mass eigenvalues

instead of mass textures, which is an approach somewhat different from what is initiated by GUT theories. It is however perfectly possible that some texture at the GUT scale translates into the observed patterns. Hence, we will point into this direction when appropriate.

The regularities in the quark and charged lepton mass spectrum can be seen in Fig. 1, where the mean values of the lepton and quark masses from Ref. [5] are plotted logarithmically over the generation number. The fact

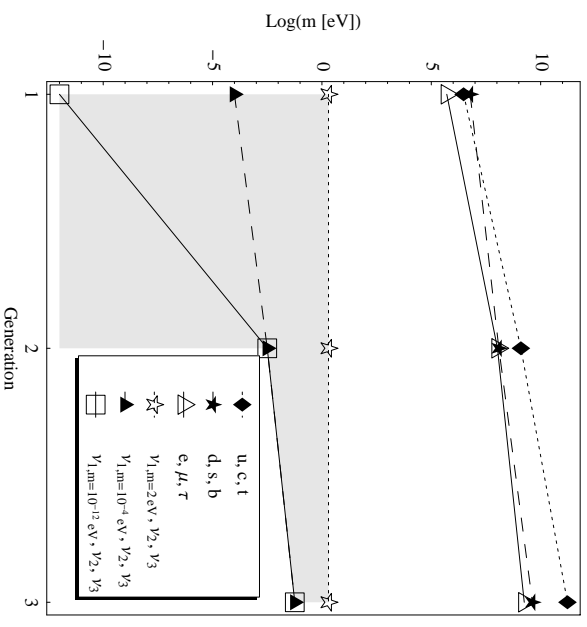


FIG. 1: Logarithm (base 10) of the quark, charged lepton, and neutrino masses plotted over the generation number. For the quark and charged lepton masses we choose the mean values given by Ref. [5]. For the neutrino masses we assume a hierarchy $m_1 < m_2 < m_3$ with the parameter values $\Delta m_{32}^2 = 3.3 \cdot 10^{-3} \text{ eV}^2$, $\Delta m_{21}^2 = 10^{-5} \text{ eV}^2$ (LMA), as well as different values for m_1 . The grey-shaded region indicates the region of allowed neutrino masses for the given LMA mass squared differences and $m_1 < 2.2 \text{ eV}$.

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that the masses lie almost on straight and parallel lines suggests that a yet unknown law describes this regularity and maybe even the “small” corrections to linearity. The linear behavior in this plot may, for example, point to an exponential or power law dependence on the generation number l . This immediately raises the question if similar regularities exist in the neutrino sector and what could be learned from these.

Neutrino masses are assumed to be generated by Dirac or Majorana mass terms in extensions of the Standard Model. In the most extreme cases, one may either have pure Dirac masses $m_D^{\nu_l}$ or pure Majorana masses $m_M^{\nu_l}$ for the physical neutrino masses, though mixtures between those are allowed in general. Depending on the model, the Majorana masses are often assumed to be created by the see-saw mechanism from Dirac masses m_D^l and heavy (right-handed) Majorana masses M_R^l , leading in the absence of leptonic mixing to the see-saw mass relation [6, 7, 8]

$$m_M^{\nu_l} = \frac{(m_D^l)^2}{M_R^l}, \quad l = 1, 2, 3. \quad (1)$$

Taking into account leptonic mixing, the Dirac mass matrix m_D can be diagonalized by two unitary matrices V_L and V_R by

$$m_D = V_L D V_R^\dagger, \quad (2)$$

where D is a diagonal matrix of the mass eigenvalues. Without loss of generality, we can choose a basis where the right-handed neutrino mass matrix M is diagonal with real and positive eigenvalues. The effective neutrino mass matrix m_ν is then given by the general see-saw relation [9]

$$\begin{aligned} m_\nu &= -m_D M^{-1} m_D^T \\ &= -\left(V_L D V_R^\dagger\right) M^{-1} \left(V_R^* D V_L^T\right). \end{aligned} \quad (3)$$

The Majorana mass eigenstates, *i.e.*, the eigenvalues of m_ν , are then entirely determined by the term $D V_R^\dagger M^{-1} V_R^* D$, which is a complex symmetric matrix, since unitary transformations do not affect the eigenvalues. Thus, if $V_R^\dagger M^{-1} V_R^*$ is approximately diagonal, *i.e.*, $V_R \simeq 1$, and the bimaximal mixings come from V_L , Eq. (1) can be used. Since we are mainly interested in qualitative mass patterns on logarithmic scales, Eq. (1) can also be used if V_R does not change the order of magnitudes of the eigenvalues significantly. The MNS matrix depends in addition on a matrix U_L^l diagonalizing the charged lepton mass matrix, *i.e.*,

$$U_{MNS} = (U_L^l)^\dagger \tilde{V}_L, \quad (4)$$

where \tilde{V}_L is diagonalizing m_ν in Eq. (3). Thus, in a basis where the charged lepton sector is diagonal, $U_{MNS} = \tilde{V}_L$. Note, however, that from U_{MNS} no direct information on V_R can be obtained, which means that mixings can enter

Eq. (3), which are depending on the neutrino mass model. We will further on initially focus on Eq. (1), which we will especially use as a tool in order to obtain small neutrino masses but not as a key ingredient of our mechanisms. Then we will discuss what this could imply for a realistic see-saw model.

We define a mass ordering $m_1 < m_2 < m_3$, *i.e.*, the mixing angles are defined correspondingly and the relations

$$m_i \geq \Delta m_{ij} \equiv m_i - m_j \quad (5)$$

with $i, j = 1, 2, 3$ and $i > j$ are satisfied in general. Thus, for given mass squared differences the neutrino mass spectrum is fixed, except from the absolute scale given by the absolute mass of one of the neutrinos. In Fig. 1, we added the information on the neutrino masses by plotting the curves for the largest allowed mass $m_1 \simeq 2$ eV and two smaller values for m_1 . Comparing the neutrino masses for different absolute mass scales m_1 with the charged lepton and quark mass hierarchies, we make the following interesting observations:

- (1) Equation (5) implies that for given mass squared differences the values of m_3 and m_2 are bounded from below, *i.e.*, for $m_1 \rightarrow 0$ $m_3 \rightarrow \Delta m_{32}$ and $m_2 \rightarrow \Delta m_{21}$. This leads to the grey-shaded region in Fig. 1, which is the region of all allowed mass spectra for the LMA solution used in the figure. The smallest values of m_2 and m_3 thus determine the steepest slope for the neutrino mass values between the generations two and three. Comparing this slope with the corresponding slopes of the charged leptons and quarks shows that they are apparently parallel. This observation suggests that there may be connections between the regularities of quark or lepton masses and neutrino masses. In addition, it points towards a hierarchical mass ordering, *i.e.*, $m_1 \leq \Delta m_{21}$.
- (2) In Fig. 1, all of the quark and charged lepton masses approximately lie on a straight line. One may assume that there is a theoretical reason for that and may thus expect the same regularity for the neutrino masses. This would imply that the neutrino masses also lie on an approximately straight line, parallel to one of the hierarchies of the other masses. Depending on what reference hierarchy is chosen, it would fix the absolute neutrino mass scale to $m_1 \simeq 10^{-5 \pm 2}$ eV, as well as it is consistent with the LMA solution.
- (3) The left-handed quarks and leptons can be symmetrically arranged in electroweak doublets. One might therefore expect that the splittings of quark and lepton masses are somehow correlated to their electroweak isospin properties, which may, for example, imply that the isospin splittings of the quarks in Fig. 1 are related to the isospin splittings of the leptons. Note, however, that the absolute

neutrino mass scale is, compared to their isospin $+1/2$ quark equivalents, shifted down by a large unknown quantity. This shifting is often believed to be done by the heavy Majorana masses M_R introduced in the see-saw mechanism in Eq. (1).

The above observations suggest that there exist empirical meaningful mass relations which indeed may allow to deduce the absolute neutrino mass scale. There are, however, different ways to combine the existing information such that different possibilities emerge. Before we will discuss some of them, let us approach the problem from a different point of view. If we assume all of the Δm^2 's to be known from measurements, we will only have to find values for one unknown parameter determining the absolute mass scale, such as m_1 . However, without any *a priori* information, we could also fit two or three of the unknown parameters m_1 , Δm_{21}^2 , and Δm_{32}^2 to their equivalents of the charged lepton and quark curves. Thus, we may distinguish three cases:

- (A) We assume Δm_{21}^2 as well as Δm_{32}^2 to be known from measurements. Then the absolute mass scale m_1 can be chosen such that the slopes of the neutrino mass curve approximately fit the slopes of one of the reference mass curves. As noted above, this leads to $m_1 \simeq 10^{-7} - 10^{-3}$ eV, depending on what hierarchy we use for reference. Thus, without additional assumptions we will not obtain preciser information. We label this case (A-2) for linking option (A) with observation (2).
- (B) We assume only one of the Δm^2 's to be known and two parameter values have to be found. We choose Δm_{32}^2 , because it is better established and measured without ambiguities. Using additional assumptions about the selection of the reference curve (*cf.*, observation (2)) or about the electroweak isospin symmetry (*cf.*, observation (3)), we can then calculate the absolute values for the masses as well as Δm_{21}^2 . The small mass squared splitting can then be used for comparison of the result with the possible solutions LMA, SMA, LOW, and VAC.
- (C) We assume none of the Δm^2 's to be known. This will not provide any information on the absolute neutrino mass scale.

We have seen that only option (B) has the potential to predict specific numerical values for the absolute neutrino masses. The simplest case is linking option (B) with observation (2), (B-2), which means that we choose a specific mass hierarchy for reference. We may, for example, assume that the physical neutrino masses are Dirac masses and directly proportional to their charged lepton partners, *i.e.*,

$$m_D^{\nu_i} = \tilde{C} \cdot m_D^l, \quad (6)$$

where \tilde{C} is some generation number-independent constant. This might also point towards a connection between the lepton masses within each electroweak isospin

doublet. The constant \tilde{C} can now be determined by the measured value of $\Delta m_{32}^2 = 3.3 \cdot 10^{-3}$ eV² from Eq. (8) in order to find $\tilde{C} \simeq 3.2 \cdot 10^{-11}$. We can then calculate the other mass squared differences $\Delta m_{31}^2 \simeq \Delta m_{32}^2$ and $\Delta m_{21}^2 \simeq 1.2 \cdot 10^{-5}$ eV² in fairly good agreement with the LMA solution. For the absolute masses we finally obtain from Eq. (6) $m_1 \simeq 1.7 \cdot 10^{-5}$ eV, $m_2 \simeq 3.4 \cdot 10^{-3}$ eV, and $m_3 \simeq 5.8 \cdot 10^{-2}$ eV, referred to as case (B-2a), which is in perfect agreement with the well-known constraints to neutrino masses. In this case, however, the smallness of \tilde{C} is not very appealing and often believed to be achieved by the see-saw mechanism, such as in Eq. (1) with the charged lepton masses for the Dirac masses m_D^l in this equation. For example, we may assume that the physical neutrino masses are Majorana masses and, for some reason, the heavy right-handed Majorana masses follow the same hierarchy as the charged lepton masses, *i.e.*,

$$M_R^l = K \cdot m_D^l, \quad (7)$$

where K is a generation number independent constant. Then we obtain for the Majorana neutrino masses from Eq. (1)

$$m_M^{\nu_i} = \frac{(m_D^l)^2}{M_R^l} = \frac{m_D^l}{K}. \quad (8)$$

Now we immediately see the connection between the case of physical Dirac neutrino masses in Eq. (6) and the case of physical Majorana neutrino masses in Eq. (8): they are mathematically equivalent for $K = 1/\tilde{C}$. It is obvious from Fig. (1) that \tilde{C} has to be very small to shift the absolute neutrino mass scale down from the charged lepton scale. This fine-tuning is often assumed to be done by the see-saw mechanism introducing the large Majorana mass scale, such as done in Eq. (7) here. Of course, Eq. (1) does not include leptonic mixings, which means that one could ask what one could learn about the mixings in the general see-saw case in Eq. (3). It is obvious from the discussion there that this see-saw mechanism would be consistent for V_R almost diagonal, *i.e.*, close to unity. This is an assumption quite often used in texture models, such as in Refs. [10, 11, 12]. In addition, other matrices V_R not affecting the order of magnitudes of the eigenvalues of m_ν could be thought about, since we consider logarithmic scales. However, our discussion does not apply to cases when the eigenvalue structure is radically changed by V_R .

Another possibility is that only the Dirac neutrino masses are related to the charged lepton masses, since these masses are produced by the same type of Yukawa couplings. Assuming the right-handed heavy Majorana mass to be universal, *i.e.*, generation index independent, we can write

$$m_M^{\nu_i} = \frac{(m_D^{\nu_i})^2}{M_R} \quad \text{with} \quad m_D^{\nu_i} = R \cdot m_D^l. \quad (9)$$

Since in this case the right-handed Majorana mass matrix M in Eq. (3) commutes with V_R , the combination

TABLE I: The different schemes as well as their results. The values for m_2 and m_3 for the scheme (A-2) are only rough estimates. The value of Δm_{32}^2 was in all cases fixed to be $\Delta m_{32}^2 = 3.3 \cdot 10^{-3} \text{ eV}^2$. The value of Δm_{21}^2 was given in the scheme (A-2) and derived in all other schemes.

Scheme	$m_1 [\text{eV}]$	$m_2 [\text{eV}]$	$m_3 [\text{eV}]$	$\Delta m_{21}^2 [\text{eV}^2]$	$\Delta m_{32}^2 [\text{eV}^2]$	Favored solution
(A-2) Rough curve comparison	$10^{-7} - 10^{-3}$	$\sim 10^{-3} - 10^{-2}$	$\sim 10^{-2} - 10^{-1}$	$1.0 \cdot 10^{-5}$	$3.3 \cdot 10^{-3}$	LMA/SMA/LOW
(B-2a) Reference curve selected (a)	$1.7 \cdot 10^{-5}$	$3.4 \cdot 10^{-3}$	$5.8 \cdot 10^{-2}$	$1.2 \cdot 10^{-5}$	$3.3 \cdot 10^{-3}$	LMA
(B-2b) Reference curve selected (b)	$4.8 \cdot 10^{-9}$	$2.0 \cdot 10^{-4}$	$5.7 \cdot 10^{-2}$	$4.2 \cdot 10^{-8}$	$3.3 \cdot 10^{-3}$	LOW
(B-3) Isospin symmetry used	$2.1 \cdot 10^{-7}$	$8.7 \cdot 10^{-4}$	$5.7 \cdot 10^{-2}$	$7.6 \cdot 10^{-7}$	$3.3 \cdot 10^{-3}$	SMA/LOW

$V_R^\dagger V_R^*$ gives the unit matrix at least in the absence of CP violation, which means that Eq. (9) is quite general. Using the same procedure as above, we obtain in this case (B-2b) for the Majorana neutrino masses $\Delta m_{21}^2 \simeq 4.2 \cdot 10^{-8} \text{ eV}^2$, $m_1 \simeq 4.8 \cdot 10^{-9} \text{ eV}$, $m_2 \simeq 2.0 \cdot 10^{-4} \text{ eV}$, and $m_3 \simeq 5.7 \cdot 10^{-2} \text{ eV}$. Note that here we do not relate the physical neutrino masses to quark or charged lepton masses and therefore do not have parallel curves.

Instead of choosing some specific mass hierarchy for reference, we may use the electroweak isospin argument from observation (3) in order to create a case (B-3). Assuming that the weak isospin $I = \pm 1/2$ lepton masses follow the same scheme as the $I = \pm 1/2$ quark masses and ignoring lepton mixings, we could postulate that

$$\frac{m_D^{\nu_l}}{m_D^l} = C \cdot \frac{m_D^{l,I=+1/2}}{m_D^{l,I=-1/2}} \quad (10)$$

with C a generation independent constant and $m_D^{l,I=\pm 1/2}$ the Dirac quark masses of generation l with weak isospin $I = \pm 1/2$. We can now again calculate the constant C , for instance, from the value of $\Delta m_{32}^2 \simeq 3.3 \cdot 10^{-3} \text{ eV}^2$, obtaining $C \simeq 8.1 \cdot 10^{-13}$. Then the values of Δm_{31}^2 and Δm_{21}^2 are determined by Eq. (10) and can be evaluated to be $\Delta m_{31}^2 \simeq \Delta m_{32}^2$ and $\Delta m_{21}^2 \simeq 7.6 \cdot 10^{-7} \text{ eV}^2$. For the absolute neutrino masses we here obtain $m_1 \simeq 2.1 \cdot 10^{-7} \text{ eV}$, $m_2 \simeq 8.7 \cdot 10^{-4} \text{ eV}$, and $m_3 \simeq 5.7 \cdot 10^{-2} \text{ eV}$. Note, however, that the weak isospin is actually defined in terms of flavor eigenstates and not mass eigenstates which we are using here. Taking into account mixings, linear combinations of neutrino masses would enter at least in $m_D^{\nu_l}$ in Eq. (10). Thus, the plausibility of this approach would strongly constrain lepton mixings or point more towards a generation index number rather than isospin dependent property.

Let us come back to our original argument in observation (2), *i.e.*, that we want to have a neutrino mass spectrum with slopes such that it looks similar to the charged lepton and quark curves. Figure 2, showing the results of our three calculations, indicates that the scheme (B-2a) fits best the other quark and charged lepton mass hierarchies. This scheme predicted a $\Delta m_{21}^2 = 1.2 \cdot 10^{-5} \text{ eV}^2$, which is in agreement with the LMA solution. Since we were only using very simple models and only one possible value for Δm_{32}^2 , some factor difference from the measured value does not destroy this conclusion.

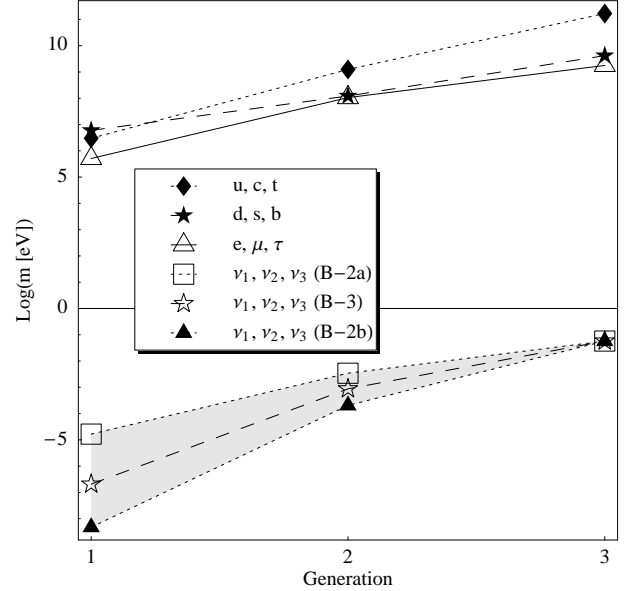


FIG. 2: Logarithm (base 10) of the quark, charged lepton, and neutrino masses obtained from our calculations plotted over the generation number. For the quark and charged lepton masses we chose the mean values given by Ref. [5]. For the neutrino masses we assumed a hierarchy $m_1 < m_2 < m_3$ with the parameter value $\Delta m_{32}^2 = 3.3 \cdot 10^{-3} \text{ eV}^2$. We obtained for the small mass squared splittings $\Delta m_{21}^2 = 1.2 \cdot 10^{-5} \text{ eV}^2$ (B-2a), $\Delta m_{21}^2 = 7.6 \cdot 10^{-7} \text{ eV}^2$ (B-3), and $\Delta m_{21}^2 = 4.2 \cdot 10^{-8} \text{ eV}^2$ (B-2b). The grey-shaded region gives an estimate for possible mass spectra from assumptions similar to the ones in this paper.

In summary, we presented very simple, purely phenomenological approaches to extract absolute values for the neutrino masses. The mass patterns which have been assumed are essentially power laws, which may arise in models when neutrino masses are generated radiatively, *e.g.*, in Froggatt-Nielsen-like models [13]. By comparing the physical neutrino mass curve plotted over the generation number with the charged lepton or quark mass curves indicated that $m_1 \simeq 10^{-7} - 10^{-3} \text{ eV}$, fixing the absolute neutrino mass scale for known mass squared differences and their signs. Using different assumptions together with simple Dirac and Majorana neutrino mass models allowed us to extract numerical values for the absolute neutrino masses in several models. As a starting

point, we used the knowledge about the neutrino mass squared differences to extract and validate these models. Table I summarizes the results from our calculations. In general, from such an approach one expects a neutrino mass spectrum between the curves of the cases (B-2a) (with linear scaling in the charged lepton masses) and (B-2b) (quadratic scaling in the charged lepton masses), indicated by the grey-shaded region in Fig. 2. All values obtained for the absolute masses agree with the well-known constraints to neutrino masses and follow, in fact, rather similar patterns. One of the most important results of such patterns would be that $m_3 \simeq 5.8 \cdot 10^{-2} \text{ eV}$ in all cases consistent with a hierarchical neutrino mass spectrum, *i.e.*, m_1 is quite small compared to the mass differences. In addition, from these purely empirical investigations the LMA solution provided the most appealing results by comparing the physical mass curves of charged lepton and neutrino masses. Moreover, the VAC solution did not fit any of our estimates. Even though such an empirical approach is by far no theory of neutrino masses, it may point to the right absolute neutrino mass scale by using the yet unexplained fermion mass patterns, *i.e.*, the patterns of mass eigenstates. We have also commented on the implications of mixings and neutrino mass models. With this method we obtained absolute neutrino masses in good agreement with all constraints, such that it could be regarded as a hint for the absolute neutrino masses. Finally, it should be interesting to investigate how the patterns in the mass eigenstates are related to textures in the mass matrices.

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